# Multidimensional stationary time series Machine Learning for Radar Clutter Classification

Yann Cabanes
*Thales LAS Limours*, France
*Institute of Mathematics of Bordeaux*, France
yann.cabanes@gmail.com

Marc Arnaudon
*Institute of Mathematics of Bordeaux*
Talence, France
marc.arnaudon@math.u-bordeaux.fr

Frédéric Barbaresco
*Thales Land & Air Systems*
6 rue de la Verrerie
F92190 Meudon, France
frederic.barbaresco@thalesgroup.com

Rami Kassab
*Advanced Radar Concepts*
*Thales LAS*
Limours, France
rami.kassab@thalesgroup.com

***Abstract*—We present a method to classify complex valued stationary centered Gaussian autoregressive time series. Our initial motivation comes from radar signal processing and especially radar clutter classification, which we detail in Section I. This issue has already been addressed in previous works, in particular by Frédéric Barbaresco [4], [7], [16], [19], [20], Le Yang [2], [28], Alice Le Brigant [9] and Alexis Decurninge [1], [13].**

**Keywords: Complex stationary centered Gaussian autoregressive time series, multidimensional time series, machine learning, Information geometry, Riemannian manifold, Block-Toeplitz matrices, Siegel disk, radar clutter, spatio-temporal correlation.**

## I. Radar motivations

The initial motivation for studying complex valued stationary centered Gaussian autoregressive time series comes from radar signal processing, in particular the study of radar clutter. In radar semantics, we distinguish between moving objects of primary interest which we call targets and information related to the radar environment which we call clutter. Radar clutter is therefore the information recorded by a radar related to seas, forests, fields, cities and other environmental elements that surround the radar. In order to better distinguish targets from clutter, it may be interesting to develop machine learning algorithms to recognize different types of clutter. Knowledge of radar clutter can be used to obtain a constant false alarm rate (CFAR) detection estimator [10], [13], [26].

To study the characteristics of the complex valued time series associated with radar clutter, it is common to assume that they are stationary centered Gaussian autoregressive time series [8]. The assumption of stationarity of the time series is here justified by extremely short observation times of the same zone of the environment. The laws of these time series are represented in Riemannian manifolds in the works of Le Yang [2], [28] and Alice Le Brigant [9]. This representation model is briefly summarized in Section II; it is applied to radar clutter clustering in the thesis work of Yann Cabanes [10].

In order to refine the study of the characteristics of radar clutter, we want to add spatial information to the temporal information contained in each time series by studying the correlation between time series recorded in spatially close cells. For this, we have developed a spatio-temporal model [10]. In order to provide the space of the coefficients of this spatio-temporal model with a Riemannian metric, the more general case of multidimensional stationary time series is studied in [10] and briefly summarized in Section III.

## II. One-dimensional stationary times series

The study of one-dimensional stationary radar time series was carried out by Frédéric Barbaresco in [3]–[7], [21]. In these works, the stationary radar time series are represented in the product space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ where $\mathcal{D}$ represents the complex unit disk. This space is endowed with a Riemannian metric inspired by information geometry. The space $\mathbb{R}_+^*$ is used to represent the average quadratic power of the studied time series. The product space $\mathcal{D}^{n-1}$ represents the coefficients of the autoregressive model, it therefore represents the Doppler information contained in the time series.

The Burg algorithm is used to estimate the coefficients of the autoregressive model from a recorded time series. This algorithm is presented in the works of Frédéric Barbaresco and Alexis Decurninge [1], [13].

The Riemannian metric constructed on the space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ is presented by Frédéric Barbaresco in [19], [22], [17], [16], [15] and related works [12], [25], [14]. This metric is also detailed in the thesis works of Le Yang [2], [28], Alice Le Brigant [9] and Yann Cabanes [10]. We refer to Shun-ichi Amari's book [27] for a full presentation of the information geometry tools used to construct this metric. We denote $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ the Riemannian manifold presented in these works: the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ corresponds to the space $\mathbb{R}_+^* \times \mathcal{D}^{n-1}$ endowed with a Riemannian metric inspired by information geometry. The computation of the mean and median in this manifold is used to detect radar targets in the work of Le Yang [2], [28]. The study of the curves of the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ is applied to the recognition of radar targets in the work of Alice Le Brigant [9]. In the thesis work of Yann Cabanes [10], the manifold $\mathbb{R}^{++} \times \mathbb{D}^{n-1}$ is used for radar clutter clustering and more generally to the classification of complex stationary centered Gaussian autoregressive time series.

## III. Multidimensional stationary times series

Complex multidimensional stationary centered Gaussian autoregressive time series are represented in a Riemannian manifold in the thesis work of Yann Cabanes [10]. As in the case of one-dimensional time series, it is possible to represent multidimensional time series by the coefficients of the autoregressive model. In the case of multidimensional time series, these autoregressive coefficients are square matrices. In the article written by Ben Jeuris and Raf Vandebril [23], the matrix coefficients of the autoregressive model are slightly modified to belong to the Siegel disk $\mathcal{SD}_N$ (set of complex matrices $N \times N$ of singular values strictly less than 1). The multidimensional stationary time series can then be represented in the space $\mathcal{H}_N^+ \times \mathcal{SD}_N^{n-1}$, where $\mathcal{H}_N^+$ is the space of Hermitian Positive Definite (HPD) matrices. This space can be endowed with a Riemannian product metric, the construction of which is detailed in the article written by Ben Jeuris and Raf Vandebril [23]. The product metric on the space $\mathcal{H}_N^+ \times \mathcal{SD}_N^{n-1}$ induces a Riemannian metric on the spaces $\mathcal{H}_N^+$ and $\mathcal{SD}_N$. The metric of the Siegel disk $\mathcal{SD}_N$ has been studied by Frédéric Barbaresco in [20], [18] and the related work [24]. The Riemannian logarithm map, the Riemannian exponential map and the sectional curvature of the Riemannian manifold defined on the Siegel space $\mathcal{SD}_N$ have been given by Yann Cabanes in [10], [11]. These geometric tools are essential for the use of certain machine learning algorithms, in particular algorithms involving a computation of the mean as the k-means algorithm.

## Acknowledgment

We thank the French MoD, DGA / AID for funding (convention CIFRE AID $N°$2017.0008 & ANRT $N°$2017.60.0062).

## References

[1] Alexis Decurninge, Frédéric Barbaresco. Robust Burg Estimation of Radar Scatter Matrix for Mixtures of Gaussian Stationary Autoregressive Vectors. *IET Radar, Sonar & Navigation, Volume 11, Issue 1*, page 78–89, January 2016.

[2] Marc Arnaudon, Frédéric Barbaresco, and Le Yang. Riemannian Medians and Means With Applications to Radar Signal Processing. *IEEE*, August 2013.

[3] Frédéric Barbaresco. Algorithme de Burg regularisé FSDS (fonctionnelle stabilisatrice de douceur spectrale) Comparaison avec l'algorithme de Burg MFE (Minimum free energy). *colloque Gretsi, Juan-les-Pins*, September 1995.

[4] Frédéric Barbaresco. Super resolution spectrum analysis regularization: Burg, Capon and AGO-antagonistic algorithms. *in EUSIPCO-96, Trieste, Italy*, pages 2005–2008, 1996.

[5] Frédéric Barbaresco. Information Geometry of Covariance Matrix: Cartan-Siegel Homogeneous Bounded Domains, Mostow/Berger Fibration and Fréchet Median. *Matrix Information Geometry; Springer*, page 199–256, 2012.

[6] Frédéric Barbaresco. Information Geometry Manifold of Toeplitz Hermitian Positive Definite Covariance Matrices: Mostow/Berger Fibration and Berezin Quantization of Cartan-Siegel Domains. March 2013.

[7] Frédéric Barbaresco, Thibault Forget, Emmanuel Chevallier, and Jesus Angulo. Doppler spectrum segmentation of radar sea clutter by mean-shift and information geometry metric. 2017.

[8] J. Barrie Billingsley. Low-Angle Radar Land Clutter, Measurements and Empirical Models. *William Andrew Publishing*, 2002.

[9] Alice Le Brigant. *Probability on the spaces of curves and the associated metric spaces using information geometry; radar applications*. PhD thesis, 2017.

[10] Yann Cabanes. *Multidimensional complex stationary centered Gaussian autoregressive time series machine learning in Poincaré and Siegel disks: application for audio and radar clutter classification*. PhD thesis, Institute of Mathematics of Bordeaux, 2022. https://hal.archives-ouvertes.fr/tel-03686052.

[11] Yann Cabanes and Frank Nielsen. Classification in the Siegel Space for Vectorial Autoregressive Data. *Geometric Science of Information*, 2021.

[12] Christian Chaire and Frédéric Barbaresco. New generation Doppler radar processing: Ultra-fast robust Doppler Spectrum Barycentre computation scheme in Poincaré's unit disk. *The 7th European Radar Conference*, pages 196–199, 2010.

[13] Alexis Decurninge and Frédéric Barbaresco. Robust Burg Estimation of Radar Scatter Matrix for Autoregressive structured SIRV based on Fréchet medians. *arXiv*, May 2016. https://arxiv.org/pdf/1601.02804.pdf.

[14] Michel Marie Deza and Elena Deza. *Encyclopedia of Distances*. Springer, 2016. https://link.springer.com/book/10.1007/978-3-662-52844-0.

[15] Frédéric Barbaresco. Modèles autorégressifs: du coefficient de réflexion à la géométrie Riemannienne de l'information. *Traitement du Signal*, 1998.

[16] Frédéric Barbaresco. Innovative tools for radar signal processing Based on Cartan's geometry of SPD matrices & Information Geometry. *IEEE Radar Conference*, 2008.

[17] Frédéric Barbaresco. Géométrie différentielle des matrices de covariance et espaces métriques à courbure négative. *GRETSI Conference, Dijon, France*, 2009.

[18] Frédéric Barbaresco. Interactions between Symmetric Cone and Information Geometries: Bruhat-Tits and Siegel Spaces Models for High Resolution Autoregressive Doppler Imagery. *In: Nielsen F. (eds) Emerging Trends in Visual Computing. ETVC 2008. Lecture Notes in Computer Science, vol 5416. Springer, Berlin, Heidelberg*, 2009.

[19] Frédéric Barbaresco. New foundation of radar Doppler signal processing based on advanced differential geometry of symmetric spaces. *International Radar Conference, Bordeaux, France*, 2009.

[20] Frédéric Barbaresco. Robust statistical Radar Processing in Fréchet metric space: OS-HDR-CFAR and OS-STAP Processing in Siegel homogeneous bounded domains. *12th International Radar Symposium (IRS)*, pages 639–644, 2011.

[21] Frédéric Barbaresco. Koszul Information Geometry and Souriau Geometric Temperature/Capacity of Lie Group Thermodynamics. *Entropy, 16(8):4521-4565*, 2014.

[22] Frédéric Barbaresco and G. Bouyt. Espace Riemannien symétrique et géométrie des espaces de matrices de covariance: équations de diffusion et calculs de médianes. *GRETSI Conference, Dijon, France*, 2009.

[23] Ben Jeuris and Raf Vandebril. The Kähler mean of Block-Toeplitz matrices with Toeplitz structured blocks. *SIAM Journal on Matrix Analysis and Applications 37(3):1151-1175*, 2016.

[24] Jérôme Lapuyade-Lahorgue and Frédéric Barbaresco. Radar detection using Siegel distance between autoregressive processes, application to HF and X-band radar. *IEEE Radar Conference*, 2008.

[25] Nicolas Charon and Frédéric Barbaresco. Une nouvelle approche pour la détection de cibles dans les images radar basée sur des distances et moyennes dans des espaces de matrices de covariance. *Traitement du Signal, Vol. 26, N°4, pp. 269-278*, 2009.

[26] F. Pascal, R. Forster, J. . Ovarlez, and P. Arzabal. Theoretical analysis of an improved covariance matrix estimator in non-gaussian noise. In *Proceedings. (ICASSP '05). IEEE International Conference on Acoustics, Speech, and Signal Processing, 2005.*, volume 4, pages iv/69–iv/72 Vol. 4, 2005.

[27] Shun-ichi Amari. *Information Geometry and Its Applications*. Springer, 2016.

[28] Le Yang. *Medians of probability measures in Riemannian manifolds and applications to radar target detection*. PhD thesis, 2011.